The e/m ratio of the electron charge to its mass

Introduction

The aim of the exercise is to determine the ratio of the charge of the electron to its mass using a magnetron method.

The path of the charged particle during its movement in the electric and magnetic fields depends on the parameters characterizing these fields, their mutual configuration and the q/m ratio of the particle's charge to its mass. A special case (often the subject of measurements by different methods) is the e/m ratio for the electron. If the field configuration and the particle path in these fields are known, the q/m value can be determined. Let's briefly consider the movement of the charged particle in the simplest situations.

In a homogeneous electric field the particle moving along the field line moves with constant acceleration. The electric force F_{el} acting on the particle is

$$\vec{F}_{el} = q \cdot \vec{E}, \tag{8.1}$$

where q - the particle charge,
E - the intensity of the electric field.

The electron, with the initial velocity equal to zero, acquires (in the field with electrical voltage U_a) the kinetic energy equal

$$eU_a = \frac{mv^2}{2}, \tag{8.2}$$

where m - resting mass of the electron,
v - the velocity acquired by the electron in the electric field,
e - the elementary charge (the electron charge).

In the magnetic field, the Lorentz force acts on a charged particle is expressed as follows:

$$\vec{F}_L = q \cdot \left(\vec{v} \times \vec{B}\right), \tag{8.3}$$

where q - the particle charge,
$\vec{v}$ - the particle velocity,
$\vec{B}$ - the magnetic field vector.

The direction of the magnetic force is perpendicular to the plane determined by the velocity and magnetic field vectors.

The Lorentz force is always directed perpendicular to the velocity of the charged particle, so it has the character of centripetal force. The magnetic field does not perform a work, but only changes the direction of the velocity of the moving particle. In a homogeneous constant magnetic field (the word homogeneous means independent of location, constant and independent on time) the particle moves on the circle when $\vec{v} \perp \vec{B}$ or more generally on the helix (if the particle has a velocity component parallel to the field direction) around the axis, which is the direction of the magnetic field vector. These cases are presented in Fig.8.1.

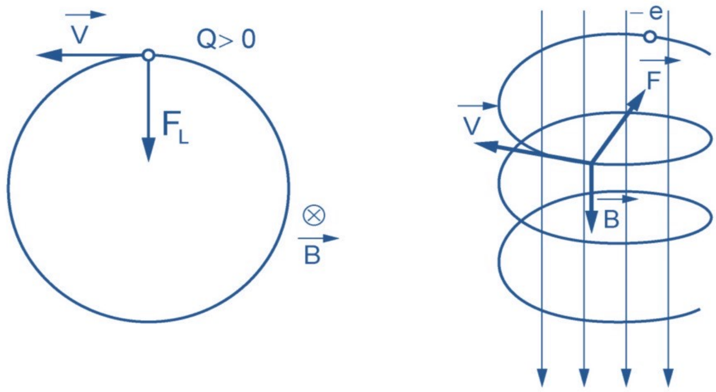

Fig. 8.1 Particle motion paths in a uniform, constant magnetic field.

In the general case, if the particle is located in the interpenetrating fields: electric and magnetic, the resultant force acting on it describes the equation:

$$\vec{F} = \vec{F}_{el} + \vec{F}_L = q\vec{E} + q\left(\vec{v} \times \vec{B}\right). \tag{8.4}$$

One of the methods for determining the charge mass ratio (e/m) of the electron is the magnetron method in which the electron is located in electric field inside the lamp and the external magnetic field. Both fields are perpendicular to each other. One of the variants of the magnetron is a vacuum diode with concentrically arranged electrodes. The anode is a metal cylinder on the axis of which there is a cathode, which is also a cylinder with a radius much smaller than the radius of the anode, or simply a thin wire placed centrally in the center of the lamp. Such a diode is placed in a homogeneous magnetic field whose lines are parallel to the axis of cylindrical electrodes (Fig.8.2).

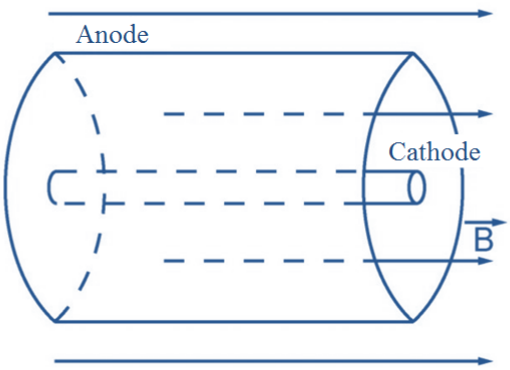

Fig. 8.2 Magnetron placed in the magnetic field - diagram.

In the absence of a magnetic field, the electrons emitted from the cathode, accelerated in the electric field due to the applied voltage U_a, move radially from the cathode to the anode (Fig. 8.3a). When both fields (electric and magnetic) act on electrons, electron motion is more complex and their paths are curved. The influence of magnetic field (at constant U_a) on the particle track is shown in Fig. 8.3b. For the determined anode voltage, there is a certain critical value of the magnetic field B_k, at which the electron tracks become tangent to the anode. For a field with the $B < B_k$ value, all the electrons sent from the cathode reach the anode and the current flowing in the magnetron has the same value as without a magnetic field. For the $B > B_k$ field, the outgoing cathode electrons do not reach the anode and the current in the lamp stops flowing.

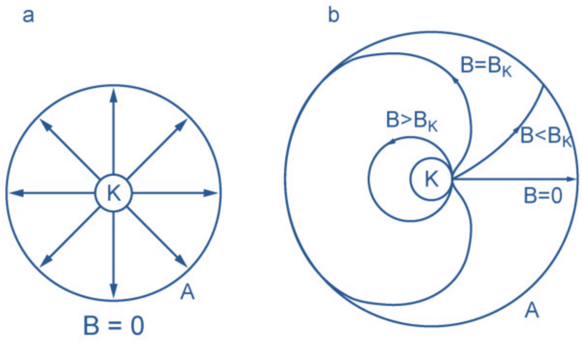

Fig. 8.3 The influence of the magnetic field on the shape of the particle track.

The dependence of anode current intensity I_a on the value of the magnetic field is illustrated by the curve in Fig. 8.4. The speed of each electron is the sum of two components - the thermal velocity and the velocity obtained in the electric field. Because the electrons emitted from the cathode have different thermal velocities, in the stream of electrons between the cathode and the anode will be both slower electrons, which at $B = B_k$ do not reach the anode and faster electrons, which reach the anode at the same magnetic field. Therefore, in diagram I_a (B) the anode current decreases gradually to values close to zero.

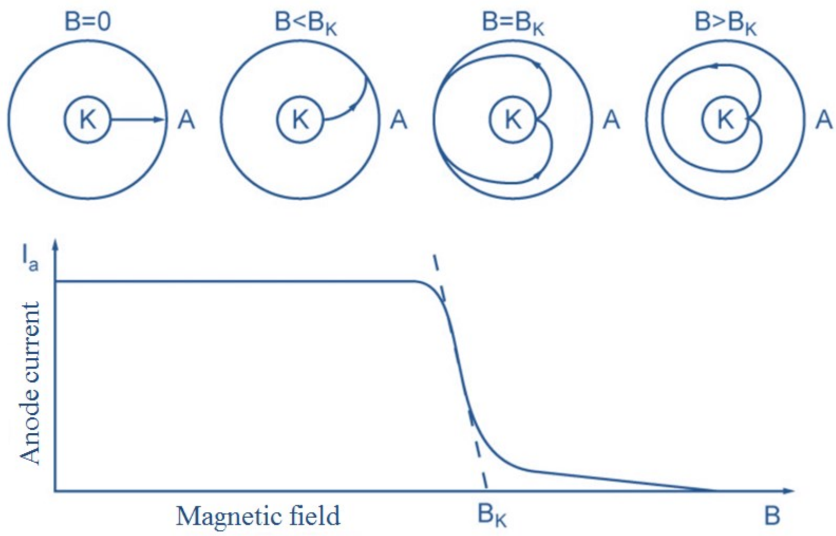

Fig. 8.4 Dependence of anode current I_a on the value of magnetic field B.

For a magnetic field with a value of B_k, when the electron's path is tangent to the circle, the radius of this circle (r) is equal to half the distance between the cathode and the anode. If we also assume that the cathode radius is much smaller than the radius of the anode, we can write that $= d/2$, where d - is the distance between the cathode and the anode. Because the velocity vectors of the electron and magnetic field are mutually perpendicular, the Lorentz force described by the dependence (8.3) is the centripetal force. When we assume that $q = e$, we get:

$$evB = m\frac{v^2}{r}. \tag{8.5}$$

After transformations of formulas 8.2 and 8.5, we get:

$$\frac{e}{m} = \frac{2U_a}{B_k^2 \cdot r^2}. \tag{8.6}$$

The dependence of 8.6 allows the calculation of e/m for the electron, if at given acceleration voltage U_a and the known r, we determine experimentally the critical value of the magnetic field B_k.

Proceeding

1. Place the lamp inside the solenoid, the coils of which are powered by a constant current. Magnetic field lines are parallel to the cylindrical axis of the lamp electrodes. The potentiometer (R_1) on the power supply is used to adjust the current flowing through the solenoid. With its help you can change the intensity of the magnetic field in the solenoid. The magnetic field B is related to the intensity of the solenoid current I_s

$$B = \mu_0 n I_s, \tag{8.7}$$

   where μ_0 - vacuum permeability ($\mu_0 = 4\pi \cdot 10^{-7}$ H/m),
   n - the number of solenoid coils per meter of length,

2. Connect the system according to the schemes placed on the laboratory table.
3. After switching the system on, wait about 5 minutes until the anode current I_a value is set.
4. Using the potentiometer (R_2) of the power supply, set the anode voltage value U_a = 4V.
5. Change the values of the solenoid current (I_s) in the range from 0 to 0.75 A and read the corresponding values of the anode current (I_a).
6. The measurements described at the 5-th item of that list should be repeated for various anode voltage values, eg U_a = 6V, 8V and 10V.
7. Save the measurement results in the table.
8. Record the number of solenoid coils per meter of length (n) and the radius (r) – half of the distance from the cathode to the anode.

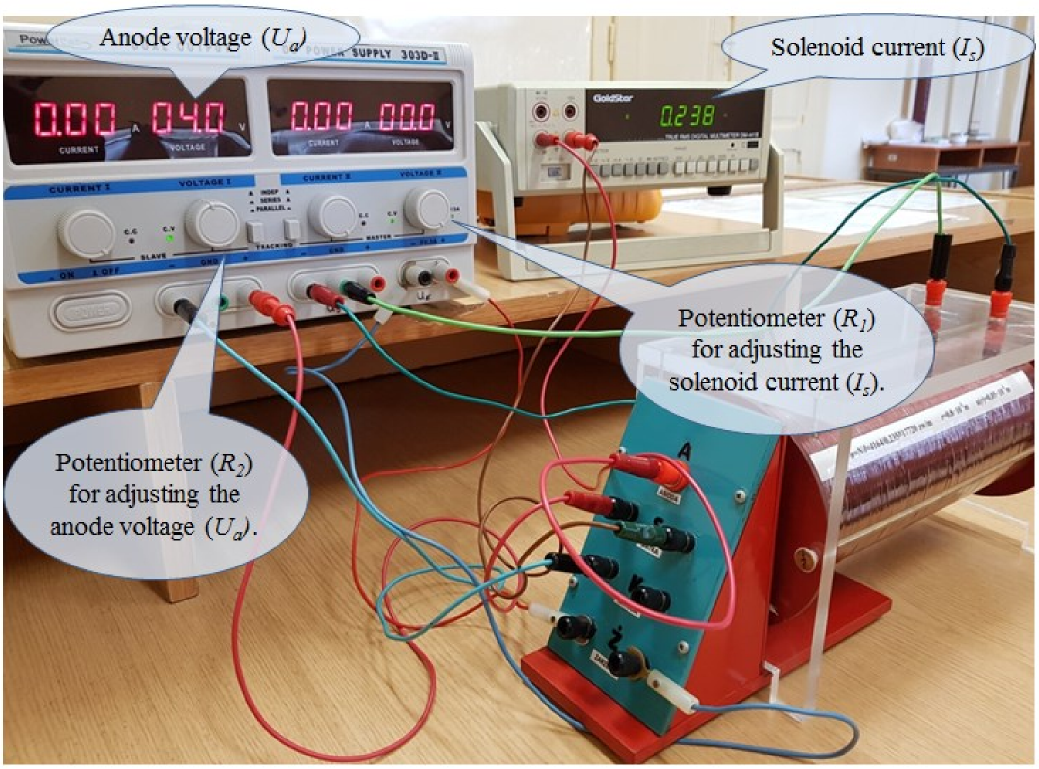

Fig. 8.5 Photo of the connected measuring set.

<u>Tables of measurements and calculation results</u>

Anode voltage							
$U_a = 4$ V		$U_a = 6$ V		$U_a = 8$ V		$U_a = 10$ V	
Solenoid current I_s (A)	Anode current I_s (μA)	Solenoid current I_s (A)	Anode current I_s (μA)	Solenoid current I_s (A)	Anode current I_s (μA)	Solenoid current I_s (A)	Anode current I_s (μA)
0.00		0.00		0.00		0.00	
0.10		0.10		0.10		0.10	
0.20		0.20		0.20		0.20	
0.27		0.30		0.30		0.30	
0.28		0.33		0.38		0.40	
0.29		0.34		0.39		0.42	
0.30		0.35		0.40		0.43	
0.31		0.36		0.41		0.44	
0.32		0.37		0.42		0.45	
0.33		0.38		0.43		0.46	
0.34		0.39		0.44		0.47	
0.35		0.40		0.45		0.48	
0.36		0.41		0.46		0.49	
0.37		0.42		0.47		0.50	
0.38		0.43		0.48		0.51	
0.39		0.44		0.49		0.52	
0.40		0.45		0.50		0.53	
0.50		0.46		0.51		0.54	
0.60		0.55		0.57		0.55	
0.70		0.65		0.66		0.65	
0.75		0.75		0.75		0.75	

Processing of the results

1. Make a graph of the dependence I_a (I_s) and read the values of I_k from the graph. For this purpose, extend the straight-line part of the curves on the graph to the intersection with the current I_s axis. The intersection point determines the value of I_k for a given value U_a (see Fig. 8.5).
2. After substituting the relation 8.7 with equation 8.6, we get an expression from which the e/m ratio for the electron can be calculated:

$$\frac{e}{m} = \frac{2}{\mu_0^2 n^2 r^2} \cdot \frac{U_a}{I_k^2}. \tag{8.8}$$

   The number of solenoid coils per meter of length $n = N/l$ and $r = d/2$.

   Calculate the ratio of e/m for each of the applied U_a voltage and next calculate the average value of $(e/m)_{av}$.

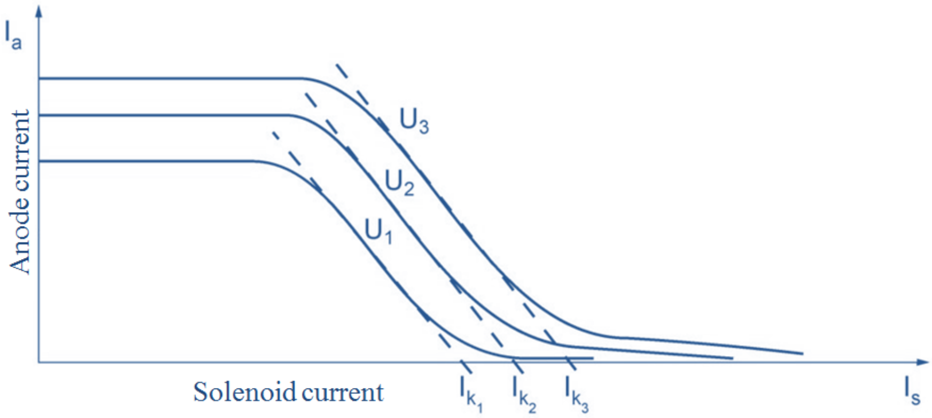

Fig. 8.5 Dependency of anode current (I_a) on the solenoid current (I_s).

3. Calculate the calibration uncertainties for directly measured quantities and estimate the uncertainty with which the values of I_k were determined.
4. Calculate the combined uncertainty $u_c(e/m)$ based on the law of the propagation of uncertainties. For ease, assume that values such as μ_0 and n are not the subject to measurement uncertainties.
5. Using the concept of expanded uncertainty, the result obtained should be compared with the reference value of e/m.

Supplementary literature

1. Andrzej Kubiaczyk, Evaluation of Uncertainty in Measurements, Warsaw University of Technology, http://www.if.pw.edu.pl …