The focal length of the lens

Introduction

The aim of the exercise is to determine focal lengths of the lenses using the optical bench.

A lens is a transparent object bounded by two spherical surfaces (convex or concave) or one spherical surface and one flat surface. The radius of the curvature of the lens is called the radius of the sphere, which is part of the limiting surface of the lens, while the center of this sphere is called the center of the curvature of the lens.

In the description of the lenses, we assume that the radii of the curvature of the convex surfaces of the lens are positive and the radii of curvature of the concave lenses are negative. The flat surface has an infinite radius of curvature. A straight line passing through the centers of curvature of both surfaces is called the central optical axis of the lens. A lens is called thin if its thickness is much smaller than the radii of curvature of the surfaces limiting the lens and we will only consider such lenses. In the thin lens, it can be concluded that the points of intersection of the central optical axis with both surfaces of the lens fall practically at the same point called the center of the lens.

A lens that causes light rays initially parallel to the central axis to converge is called a converging lens. If, instead, it causes such rays to diverge, the lens is diverging. The rays running parallel to the central optical axis, after passing through the converging lens, concentrate at one point (*F*) called the focal point (or focus) (Fig. 7.1a). The virtual focal point of the diverging lens is determined by the backward extensions of the rays dispersed by the lens (Fig. 7.1b). Each lens has two foci located at equal distances on both sides of the lens.

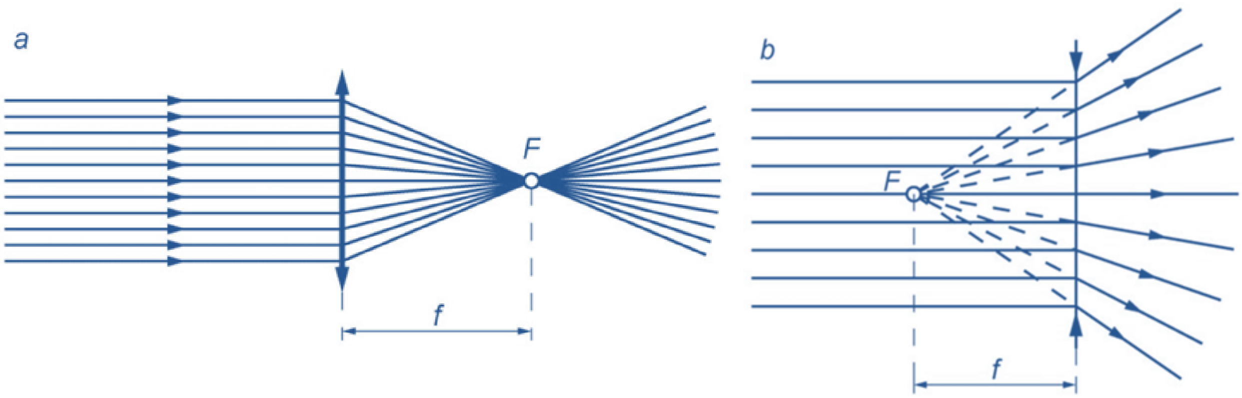

Fig. 7.1 The course of rays parallel to the central optical axis after passing through the converging (a) and diverging (b) lens.

The distance (*f*) of the focal point from the center of the lens is called the focal length of the lens. The focal length of the lens is given by the formula:

$$\frac{1}{f} = \left(\frac{n_l}{n_m} - 1\right)\left(\frac{1}{r_1} - \frac{1}{r_2}\right), \quad (7.1)$$

where f - focal length,

r_1, r_2 - the radii of the curvature of the lens,

n_l - refractive index of the lens material,

n_m - the refractive index of the medium in which the lens is located.

For converging lenses $f > 0$, for diverging: $f < 0$. From the formula (7.1) it follows that the lens type (converging, diverging) is determined by the lens geometry (r_1, r_2), the type of material (n_l) the lens is made and medium (n_m) in which the lens is placed.

To construct the image of the object given by the lens, it is sufficient to draw two of the three rays coming from one point of the object:

1. the ray passing through the optical center of the lens. It does not refract (we omit a slight, parallel shift),
2. a ray parallel to the main optical axis which, after the refraction in the lens, passes through the focus F,
3. the ray passing through the focus F, which after the refraction in the lens runs parallel to its optical axis.

With the use of converging lenses, we get real images (at the intersections of rays) or virtual images (at the intersections of the rays' extensions) (Fig. 7.2). Divergent lenses allow only the virtual image of an object to be obtained.

Linear magnification m of the image is called the ratio of the linear dimensions of the image to the linear dimensions of the object (see Fig. 7.2).

$$m = \frac{A'B'}{AB} = \frac{y}{x}\ , \quad (7.2)$$

where x - the distance of the object from the lens,

y - the distance of the image from the lens.

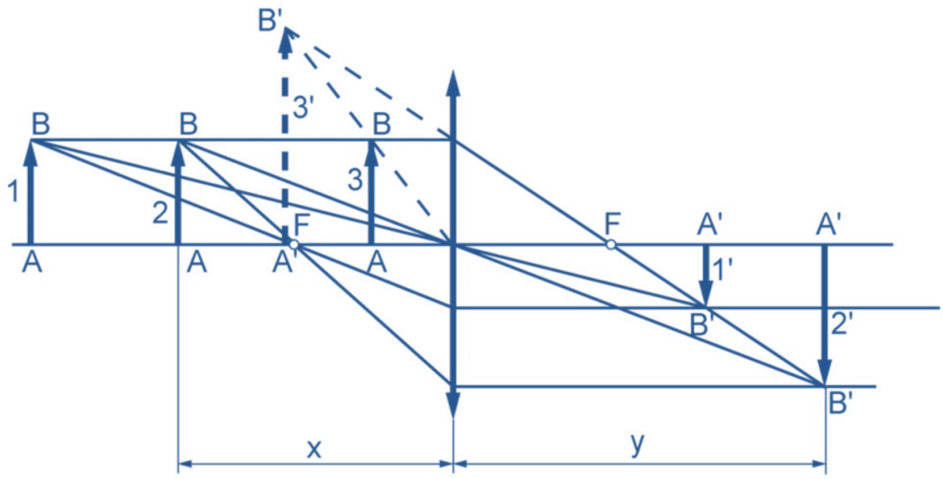

Fig. 7.2 The course of rays and the construction of images obtained in a converging lens.

Between the distance x of the object from the lens, the distance y of the image from the lens and the focal length f of the lens there is the following relationship called the lens equation:

$$\frac{1}{f} = \frac{1}{x} + \frac{1}{y}. \tag{7.3}$$

If a real image is obtained by means of the lens, the distance of the object from the image fulfills the condition

$$x + y = d \geq 4f. \tag{7.4}$$

The greater effect a lens has on light rays, the more powerful it is said to be. The power P of the lens is defined to be the inverse of its focal length:

$$P = \frac{1}{f}. \tag{7.5}$$

Dioptre is a unit of measure of the optical power of the lens, which is equal to the reciprocal of the focal length measured in meters.

In practice, systems consisting of several lenses are often used. It can be shown that in the case of thin, close-lying lenses, the optical power $1/f$ is equal to the algebraic sum of the optical powers of its individual lenses.

$$\frac{1}{f} = \sum_n \frac{1}{f_n}. \tag{7.6}$$

Proceeding

The focal length of the lens can be determined by various methods.

1. Measurement of the distance of the object and image from the lens.

From equation (7.3) we get:

$$f = \frac{xy}{x+y}. \tag{7.7}$$

To determine the focal length of the lens you need to measure x and y. To do this:

a) Place a shining object on the optical bench, a screen at a distance of about 1 m from the object.
b) Place a converging lens between the object and the screen.
c) Move the lens along the optical bench to obtain a clear, sharp image of the object on the screen. Measure the x and y distances.
d) Measure the x and y for three different distances between the shining object and the screen.

2. Determination of the focal length of the lens from the size of the enlarged image.

From the formulas (7.2) and (7.3) the following dependence on the focal length of the lens can be deduced:

$$f = \frac{yL}{L+L'}, \tag{7.8}$$

where L - the size of the object (the height of the shining arrow),
L' - the size of the image,
y - the distance of the image from the lens.

a) Move the lens along the optical bench to obtain a sharp (enlarged) image of the object on the screen.
b) Measure the size of the object L, the size of the image L' and the distance of the image from the lens y.
c) Measure the quantities described above three times, for three different distances between the shining object and the screen.

Note that the formula (7.8) is also true for the reduced image. The value f determined from the measurements for the reduced image, due to the smaller values of L' and y, will be subject to greater uncertainty compared to the value f obtained from the enlarged image. Therefore, it is not recommended to set the focal length of the lenses from the size of the reduced image.

3. Determination of the focal lens using the Bessel method.

If the distance of the object from the screen is marked as d and we take into account that $d = x + y$, the equation 7.3 can be formulated as follows:

$$\frac{1}{f} = \frac{1}{x} + \frac{1}{d-x}, \tag{7.9}$$

and after small transformations:

$$x^2 - dx + df = 0. \tag{7.10}$$

It is a quadratic equation with respect to x. The solutions of this equation are two x values described as follows:

$$x = \frac{d-\sqrt{d^2-4df}}{2}, \qquad x = \frac{d+\sqrt{d^2-4df}}{2}. \tag{7.11}$$

It seems that at a fixed distance d there are two positions of the lens, at which a sharp image of the object will arise. Both of these solutions make sense when $d^2 - 4df > 0$, i.e. $d > 4f$. When $d = 4f$ there is only one lens position at which a sharp image will appear ($x = y = 2f$). In the Bessel method, we tend to obtain two lens positions corresponding to sharp images at constant d, i.e. $d > 4f$. The distances of the object from the lens are equal to x and x', respectively.

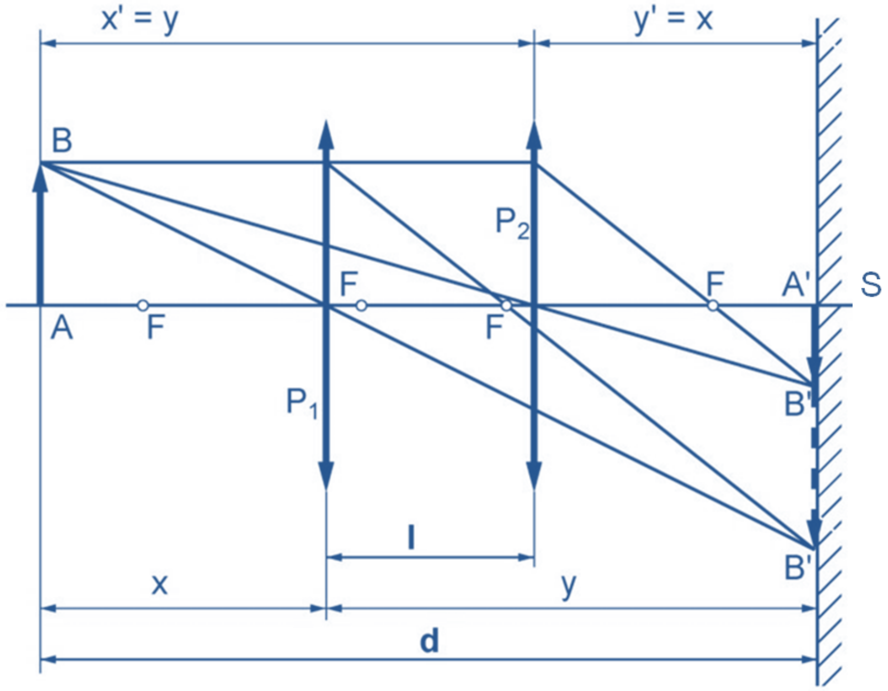

Fig. 7.3 Scheme of the Bessel method. P_1 and P_2 - two positions of the lens corresponding to the "sharp" image of the object A'B 'on the screen S.

Marking the distance between these two lens positions (Fig. 7.3) as l and considering (7.11) we will get:

$$l = x' - x, \tag{7.12}$$

$$l = \sqrt{d^2 - 4df}, \tag{7.13}$$

$$f = \frac{d^2 - l^2}{4d}. \tag{7.14}$$

As it results from dependence (7.14), to determine the focal length of the lens:

a) Position the object and screen at the opposite ends of the optical bench. Measure the value of d.
b) Move the lens along the optical bench, finding two positions corresponding to the sharp image of the object, and determine the distance between the sharp images (l).
c) Do the measurements for two different d values.

The Bessel method is the most accurate of the described methods. It is particularly useful for determining the focal length of thick lenses and lens systems, because in this method we do not need to know the exact position of their optical center.

4. Determination of the focal length of diverging lens

Measuring the focal length of the divergent lens directly using one of the described above methods is impossible, because the divergent lens does not give real images. In order to determine the focal length f_1 of the divergent lens, it should be combined with a convergent lens of known focal length f_2, so that the created system is a focusing system ($|f_2| < |f_1|$).

The ability of focusing of a system of thin lenses having a common optical axis is expressed by the formula:

$$\frac{1}{f_{1,2}} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{h}{f_1 f_2}, \tag{7.15}$$

where $f_{1,2}$ - focal length of the lens system,

h - the distance between the centers of the lenses.

From where:

$$f_1 = \frac{f_{1,2}(f_2 - h)}{f_2 - f_{1,2}}. \tag{7.16}$$

When determining the focal length of the lens system ($f_{1,2}$), proceed in the same way as in the Bessel method described above (3a, 3b, 3c).

Knowing the previously determined focal length (f_2) of the converging lens and the focal length of the system ($f_{1,2}$) from the relationship (7.14), the calculation of the focal length of the diverging lens (f_1) can be done using 7.16 formula.

Tables of measurements and calculation results

Converging lens				
Number	x (cm)	y (cm)	f (cm)	f_{av} (cm)
1				
2				
3				

Converging lens					
Number	y (cm)	L (cm)	L' (cm)	f (cm)	f_{av} (cm)
1					
2					
3					

Converging lens				
Number	d (cm)	l (cm)	f (cm)	f_{av} (cm)
1				
2				

Lens system				
Number	d (cm)	l (cm)	$f_{1,2}$ (cm)	$f_{(1,2)av}$ (cm)
1				
2				

Processing of the results

1. Based on the formula 7.7 calculate the focal length (f) for each pair of the x and y measurements. Next calculate the average value (f_{av}) of the focal length of the converging lens.
2. Calculate the uncertainties u(x) and u(y) based on the estimated values of the calibration uncertainty and the investigator uncertainty. Based on them, calculate the combined uncertainty $u_c(f)$ using the law of propagation of uncertainty.
3. Based on the formula 7.8 calculate the focal length (f) for each of the y, L and L' measurements. Next calculate the average value (f_{av}) of the focal length of the converging lens.
4. Calculate the uncertainties u(y), u(L) and u(L') based on the estimated values of the calibration uncertainty and the investigator uncertainty. Based on them, calculate the combined uncertainty $u_c(f)$ using the law of propagation of uncertainty.
5. Calculate the focal length of the lens using formula 7.14 for each pair of the d and l measurements. Next calculate the average value (f_{av}) of the focal length of the converging lens.
6. Calculate the uncertainties u(d) and u(l) based on the estimated values of the calibration uncertainty and the investigator uncertainty. Based on them, calculate the combined uncertainty $u_c(f)$ using the law of propagation of uncertainty.
7. Calculate the focal length of the lens system using formula 7.14 for each pair of the d and l measurements. Next calculate the average value of the focal length of the lens system.
8. Knowing $f_{1,2}$ and f_2, calculate the focal length of the diverging lens using formula 7.16.
9. Similarly to the previous uncertainty, calculate the combined uncertainties of the focal length of the system and the diverging lens.
10. Compare focal lengths for the converging lens, determined by different methods. Check if the same results were obtained within the limits of uncertainty.

Supplementary literature

1. Andrzej Kubiaczyk, Evaluation of Uncertainty in Measurements, Warsaw University of Technology, http://www.if.pw.edu.pl …