Young’s modulus

Introduction

The aim of the exercise is to determine the Young's modulus for different materials.

External forces acting on the body can cause (apart from the body movement in accordance with the principles of dynamics) also its deformation. The action of external forces causing deformation (change in the volume or shape of the tested body) is accompanied by disturbance of the intermolecular forces distribution, manifested by the appearance of internal forces called elastic forces.

Deformation of a solid body under the influence of external forces is accompanied by the movement of these body particles from the original balance position to the other. The interaction between molecules counteracts this. If the displacements of the particles are not too large, then after the disappearance of external forces, the particles return to their original equilibrium positions due to internal forces. This type of deformation is called elastic deformation.

If internal forces, after disappearance of external forces, are not capable of restoring the molecules to the original position, then this deformation is called plastic. Often, however, if the action of even small external forces is long-lasting, elastic deformation can become a plastic deformation. This is due to the change in the structure of the crystal lattice of the solid body under the influence of long-lasting forces.

In the case of elastic deformations, the physical quantity, equal in number to the elastic force (F) per unit of the cross-section of the body (S), is called the stress (σ).

$$\sigma = \frac{F}{S}. \tag{6.1}$$

In a situaltion where the acting force is directed along the normal to the S surface, we speak of normal stress or tensile stress.

Derived from the experiments by an English physicist R. Hooke, the law says:

The stress σ of the elastically deformed body is linearly proportional to its fractional extension or strain ε.

$$\sigma = \frac{1}{k}\varepsilon, \tag{6.2}$$

where k - elasticity coefficient depending on the material of which the body is made (for different types of deformation it adopts different names, designations and numerical values),

ε - relative deformation,

σ - stress.

All the deformations of the body can be divided into three main groups:

- tensile tension or compression,
- comprehensive compression or stretching known as uniform hydraulic stress,
- shearing stress.

For tensile stress, the Hooke's law takes the following form:

$$\sigma = E \cdot \frac{\Delta l}{l_0}, \tag{6.3}$$

where: E – proportionality factor called the Young's modulus,
$\Delta l/l_0$ – fractional change in a length of the specimen.

Young's modulus can be imagined as a stress inducing a length increase equal to the initial length. It fits the rubber cord well, but it hardly corresponds to stretching of steel.

In order to determine the value of the Young's modulus, we use the method based on the use of the complex deformation that occurs during the bending of a rod supported in two places near its ends (Figure 6.1).

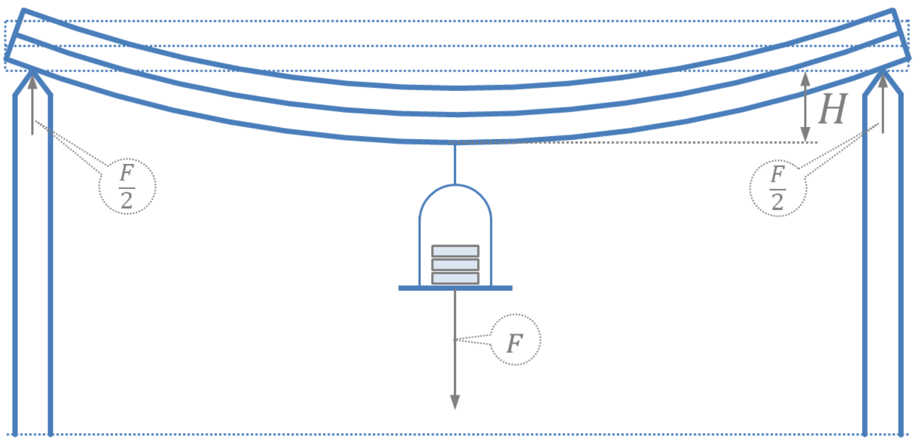

Fig. 6.1 Deformation of the rod based on two supports.

Bending the body can be considered as simultaneous squeezing of the upper and stretching of the lower body surface (Figures 6.1, 6.2).

The measure of deformation is the deflection (usually called deflection arrow) (H). According to Hooke's law, the deflection arrow (H) for a rod supported freely on two sides is proportional to the deforming force (F) and can be expressed by the following formula:

$$H = \frac{l^3}{4ah^3} \cdot \frac{F}{E}, \tag{6.4}$$

where: a – width of the rod,
h – rod thickness measured in the direction of force F,
l – rod length understood as the distance between the rod's support points.

By converting this equation, we can determine the Young's modulus

$$E = \frac{l^3}{4ah^3} \cdot \frac{F}{H}. \tag{6.5}$$

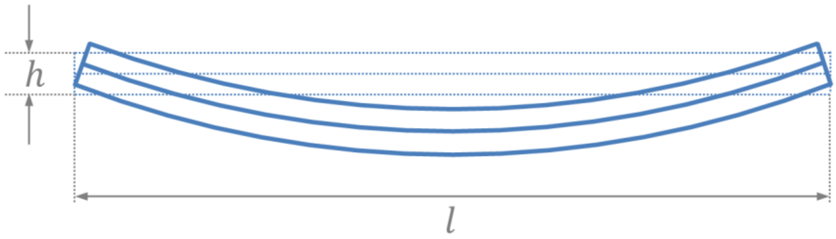

Fig. 6.2 View of the bent rod with its dimensions.

Proceeding

1. Using a vernier caliper measure the cross-sectional edges a and h of the tested rod. The measurement should be made at three different points of the rod.
2. Place the tested rod on supports in such a way that the support points are located about 1 cm from the ends of the rod.
3. Measure the rod length l (the distance between the rod's support points).
4. On the center of the rod place the stirrup with the pan.
5. Adjust the position of the micrometer so that the measuring element contacts the upper part of the stirrup and has the ability to indicate the deflection arrow when loading the rod.
6. Place gradually increasing weights ($F = mg$) on the pan and note the deflections H_1 for each load, then note the rod deflections (H_2) in the load reduction for each of the load.
7. Using the balance with digital display, weigh all the weights that were used to load the rod.
8. The measurement procedure described in items 1 to 7 should be repeated for two other rods made of different materials.

Tables of measurements and calculation results

The material the rod is made of	Length l (cm)	Edge a (mm)	Average edge $\bar{a}$ (mm)	Edge h (mm)	Average edge $\bar{h}$ (mm)

The material the rod is made of	Number	Mass m (kg)	Force F (N)	H_1 (mm)	H_2 (mm)	$\overline{H}$ (mm)	$F/\overline{H}$ (N/mm)	$\overline{F/\overline{H}}$ (N/m)	Young's modulus E (N/m^2)
	1								
	2								
	3								
	4								
	5								
	6								
	1								
	2								
	3								
	4								
	5								
	6								
	1								
	2								
	3								
	4								
	5								
	6								

Processing of the results

1. Based on the formula $F = mg$ calculate the load on the rod and save it in the table.
2. Calculate the deflection arrow $\overline{H}$ as an average value from the deflections H_1 and H_2 .
3. Calculate the ratio F to $\overline{H}$, and then average its value $\overline{F/\overline{H}}$.
4. Based on the formula:

$$E = \frac{l^3}{4 \cdot \overline{a} \cdot \overline{h}^3} \cdot \overline{\left(\frac{F}{\overline{H}}\right)}. \tag{6.6}$$

   calculate the Young's modulus.

5. The uncertainty of the Young's modulus should be determined by using the concept of combined uncertainty for uncorrelated quantities.
6. Compare the obtained Young's modules with reference values and check if your results are consistent with the reference ones, taking into account the concept of expanded uncertainty.

Supplementary literature

1. Andrzej Kubiaczyk, Evaluation of Uncertainty in Measurements, Warsaw University of Technology, http://www.if.pw.edu.pl …