Hall effect

Introduction

The aim of the exercise is to study the Hall phenomenon, and in particular, determination of the Hall constant and the concentration of electrical current carriers.

In 1879 E.H. Hall planned an experiment to determine the sign of current carriers moving within the semiconductor, as well as their concentration and mobility. Let's follow this reasoning.

Let an electric current of density j (Fig. 2.1) flow through the semiconductor, which has the shape of a rectangular plate. In this case, the current density vector coincides with the direction of the electric field applied to the sample. If the semiconductor is homogeneous, then the equipotential plane passing through ac (Fig. 2.1), perpendicular to the direction of the electric field E, is also perpendicular to the vector of current density j. Therefore, the electrical potential difference between points a and c is equal to zero. We will now place the semiconductor in a homogeneous magnetic field (B) whose lines are perpendicular to the direction of current flow (see Fig. 2.1).

The Lorentz force is acting on the electric charge Q, moving at the velocity v in the magnetic field B, in accordance with the formula:

$$\vec{F} = Q\left(\vec{v} \times \vec{B}\right). \tag{2.1}$$

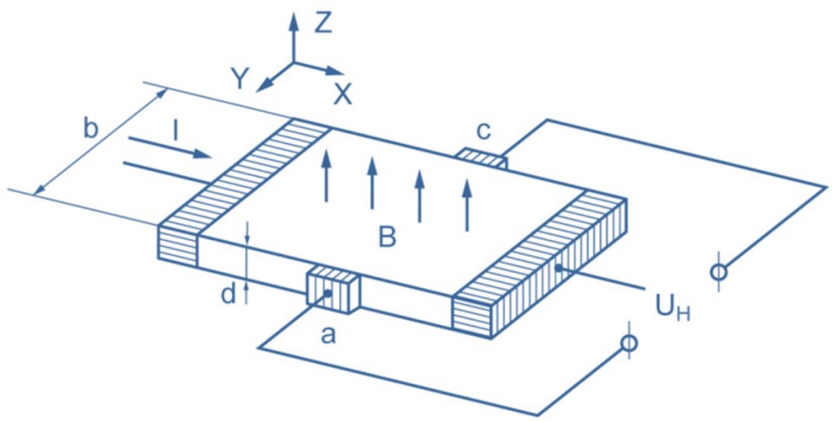


Fig. 2.1 Hall voltage measurement system

The direction of this force depends on the sign of the charge carriers Q and the vector product of the velocity v and magnetic field B. If the velocity of the charge carriers has a component perpendicular to the magnetic field B, then under the action of the Lorentz force there is a deflection of the charge carriers in the direction perpendicular to v and B. The result is a spatial separation of charge carriers and the electric field E_H appears (Figure 2.2)

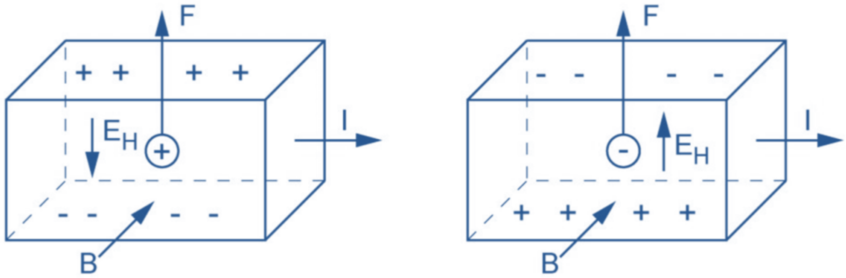


Fig. 2.2 The deflection of the direction of movement of the current carriers under the influence of the magnetic field in the samples with hole (a) (on the left side) and electron conductivity (b) (on the right side of the figure).

The consequence of the charge separation is the appearance of the difference in electrical potentials U_H between the ac points (see Fig 2.1). This effect is called the Hall effect, and the appearing difference in electrical potentials U_H - Hall's voltage.

Under the influence of Lorentz force, for the given direction of B and E, the holes in the acceptor semiconductor (Fig. 2.2a) and electrons in the donor semiconductor (Fig. 2.2b) decline towards the upper wall of the sample, and there is a deficit of them on the bottom wall. It results in the creation of opposite potential on the upper and lower wall respectively. This process continues as long as the Hall electric field generated by the separation of charge carriers does not create a force acting on free load carriers balancing the Lorentz force. In the state of equilibrium, these forces are equal in number and for the electron semiconductor they meet the equality:

$$eE_H = evB. \tag{2.2}$$

If the width of the sample is b, the thickness d, then the potential difference U_H is:

$$U_H = E_H b = -vBb. \tag{2.3}$$

The current I, the current density j and the speed of the carriers v meet the dependencies:

$$I = jS, \quad S = bd, \quad j = env \tag{2.4}$$

When determining v from the above equations, the expression (2.3) can be written:

$$U_H = -\frac{1}{en} \cdot \frac{1}{d} \cdot IB = R \cdot \frac{B}{d} \cdot I. \tag{2.5}$$

The value of R in equation (2.5) is called the Hall constant and in the case of electrons it equals to:

$$R = -\frac{1}{en}. \tag{2.6}$$

If the charge carriers are holes with a concentration p, then taking into account Fig. 2.2a, the equation (2.5) will take the form:

$$U_H = \frac{1}{ep} \cdot \frac{1}{d} \cdot IB = R \cdot \frac{B}{d} \cdot I. \tag{2.7}$$

And the Hall constant

$$R = \frac{1}{ep}. \tag{2.6a}$$

As can be seen from the equation (2.5), the measurement of Hall's voltage U_H enables the calculation of Hall's constant (2.5a) if the values of magnetic field B, current intensity I and sample thickness d are known.

$$R = \frac{d \cdot U_H}{B \cdot I}. \tag{2.5a}$$

It, in turn, gives the possibility to calculate the concentration of charge carriers n from dependence (2.6).

The formula (2.5) can also be used to determine the magnetic field B. In modern information and multimedia technology, hallotrons (semiconductors used to measure the magnetic field B using the Hall effect) are widely used.

The laboratory setup

The measuring setup consists of a hall sensor (semiconductor plate) located in the gap between the magnets and a mechanical system equipped with a knob with an angular scale, ensuring the rotation of the magnets around the plate. The magnets produce a constant and homogeneous field of known value (B_0 = 1 T). The rotation of the magnets make of changing the direction of the magnetic field vector B in relation to the main axis of the hallotron plate along which the current I_x flows (see fig. 2.1). The setup includes sockets for connecting the meters: voltmeter for U_H measurements and ammeter for I_x measurements. The source of the current I_x is a stabilized power supply enabling smooth change of the current I_x.

After turning on the power with the switch located in the rear part of the set housing, it is required to wait about 3-5 minutes.

The setup is shown in the photo below.

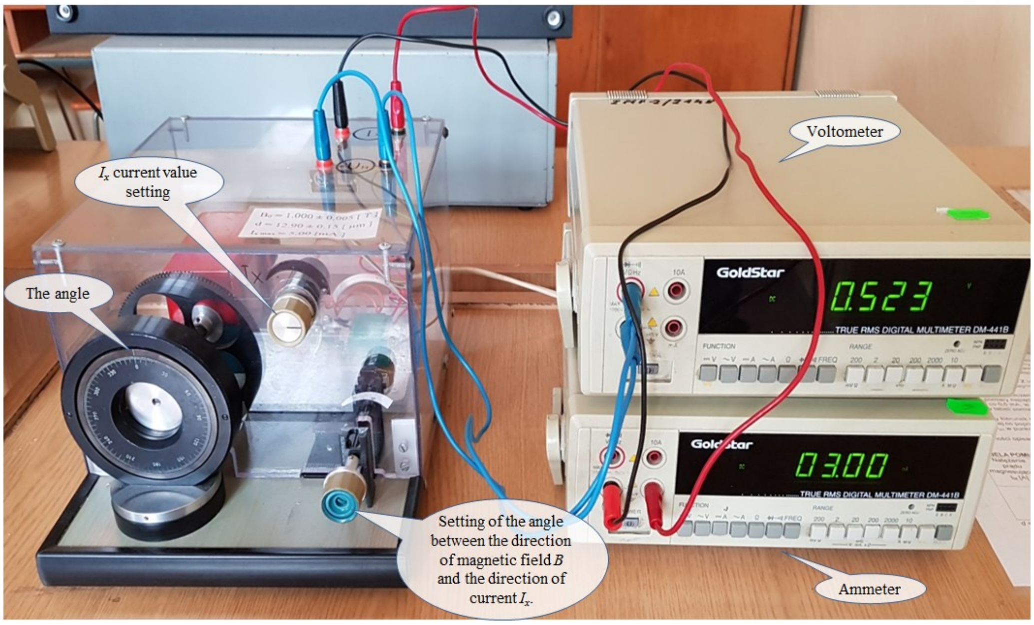


Fig. 2.3 Photograph of the Hall effect setup.

Proceeding

1. Connect the meters for measuring the current I_x and the Hall voltage U_H. Set the minimum value with the I_x current adjustment knob. In every meter, use the buttons located on the front panel, activating the appropriate work function (voltmeter or ammeter) and the corresponding measuring range (see fig. 2.3).
2. After checking the connections by the teacher, turn on the meters and then power on the system.
3. Use the knob located on the front panel of the system to set the angle α. At start set the value of $\alpha = 0$. For this angle the direction of the vector of magnetic field B and the direction of the I_x are at right angle.
4. From $I_x = 0$ mA, gradually increase the current, with the step of 0.5 mA until the 5 mA is reached. Save the readings of the I_x and U_H meters. Assume the Hall voltage (U_H) as positive even if the meter shows negative. Do the same for currents.
5. Change the angle of the magnets by 180^0 (In this case, the direction of the magnetic field is opposite to the one set out in point 3).
6. For the newly set value of the angle α, perform measurements according to point 4.
7. Under the Results Table, save the information relevant for estimating the uncertainty of the measurements made. Write down the information about the constants of the system (located on the casing of the system; (currently: B_0 = (1.000 ± 0.005) T, d = 12.90 ± 0.15) µm, $I_{x\ max}$ = 5 mA)).
8. After taking measurements, turn the I_x current knob to minimum.

The Results Table

α	I_x	$U(I_x)$	U_H	$U(U_H)$
[^{0}]	[mA]	[mA]	[V]	[V]

Development of measurement results

1. For each value of I_x, calculate the expanded uncertainty U(I_x). Take k = 2. Record the results in the table above.

$$U(I_x) = k \cdot u(I_x) = k \cdot \sqrt{\frac{\left(\Delta_e(I_x)\right)^2 + (C_1 \cdot I_x + C_2)^2}{3}}. \qquad (2.8)$$

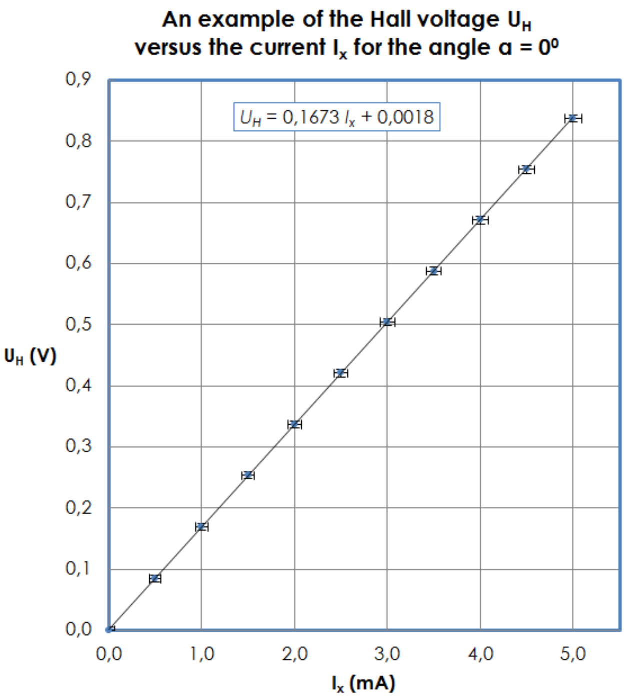
   The values of C_1 and C_2 for the used meter (DM-441B) are respectively: C_1 = 0.005, C_2 = 1μA. If, when reading the current I_x, the value displayed on the meter changed slightly, then the maximum of such a change can be treated as the factor influencing on the investigator's uncertainty $\Delta_e(I_x)$. The assumption that $\Delta_e(I_x)$ = 0.010 mA will allow for displaying the visible uncertainty bars on the graph of the dependence of U_H = f (I_x).

2. Make estimation/calculation of the expanded (k = 2) standard uncertainty $U(U_H)$ according to the following formula:

$$U(U_H) = k \cdot u(U_H) = k \cdot \sqrt{\frac{\left(\Delta_e(U_H)\right)^2 + (C_1 \cdot U_H + C_2)^2}{3}}. \qquad (2.9)$$

   The constants for the used DM-441B are as follows: C_1 = 0.001, C_2 = 400 μV. As in the case of currents, the investigator's uncertainty can also be assumed in the case of the measured voltage. If you have no other idea, take $\Delta_e(U_H)$ = 5.0 mV.

3. Make a graph of the Hall voltage U_H versus the I_x current. Plot uncertainty bars on the chart. Draw a trend line, which should be a straight line that best fits the measurement points.



4. Using the LINEST (Excel) function, determine the values of the parameters of the equation of a straight line ($U_H = a \cdot I_x + b$). In the table below write down the values of a and b, their uncertainties and their units.

Slope of the line ($U_H = a \cdot I_x + b$)		Intercept of the line ($U_H = a \cdot I_x + b$)	
a	$u(a)$	b	$u(b)$
[complete the unit]	[complete the unit]	[complete the unit]	[complete the unit]

5. Calculate the value of the Hall constant for the semiconductor hallotron used in the exercise:

$$R = \frac{a \cdot d}{B_0}. \tag{2.10}$$

6. Calculate the expanded (k = 2) standard uncertainty $U(R)$ according to the following formula:

$$U(R) = k \cdot R \cdot \sqrt{\left(\frac{u(a)}{a}\right)^2 + \left(\frac{u(d)}{d}\right)^2 + \left(\frac{u(B_0)}{B_0}\right)^2}. \tag{2.11}$$

7. Calculate the concentration (n) of charge carriers in the tested semiconductor:

$$n = \frac{1}{e \cdot R}. \tag{2.12}$$

When the charge carriers are electrons, e can be understood as an elementary charge of $1.602 \cdot 10^{-19}$ C.

8. Calculate the expanded (k = 2) standard uncertainty $U(n)$ according to the following formula:

$$u(n) = k \cdot n \cdot \frac{u(R)}{R}. \tag{2.13}$$

9. Formulate conclusions for the exercise.